



## Cambridge International AS & A Level

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NAME

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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3

**February/March 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.



**1** It is given that  $x = \ln(2y - 3) - \ln(y + 4)$ .

Express  $y$  in terms of  $x$ .

[3]

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## 3

- 2 (a) On an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $-\frac{1}{3}\pi \leq \arg(z - 1 - 2i) \leq \frac{1}{3}\pi$  and  $\operatorname{Re} z \leq 3$ . [3]

- (b) Calculate the least value of  $\arg z$  for points in the region from (a). Give your answer in radians correct to 3 decimal places. [2]

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- 3 The polynomial  $2x^4 + ax^3 + bx - 1$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . When  $p(x)$  is divided by  $x^2 - x + 1$  the remainder is  $3x + 2$ .

Find the values of  $a$  and  $b$ . [5]

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4 Solve the equation

$$\frac{5z}{1+2i} - zz^* + 30 + 10i = 0,$$

giving your answers in the form  $x + iy$ , where  $x$  and  $y$  are real.

[5]

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5 The parametric equations of a curve are

$$x = te^{2t}, \quad y = t^2 + t + 3.$$

(a) Show that  $\frac{dy}{dx} = e^{-2t}$ . [3]

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(b) Hence show that the normal to the curve, where  $t = -1$ , passes through the point  $(0, 3 - \frac{1}{e^4})$ . [3]

Dotted lines for writing the answer.





(b) Hence solve the equation  $5 \sin 2x + 12 \cos 2x = 6$  for  $0 \leq x \leq \pi$ . [4]

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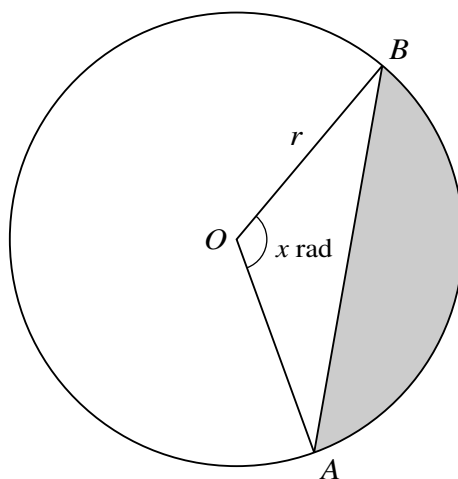
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The diagram shows a circle with centre  $O$  and radius  $r$ . The angle of the **minor** sector  $AOB$  of the circle is  $x$  radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that  $x = \frac{3}{4} \sin x + \frac{1}{2}\pi$ .

[4]

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(b) Show by calculation that the root of the equation in (a) lies between 2 and 2.5. [2]

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(c) Use an iterative formula based on the equation in (a) to calculate this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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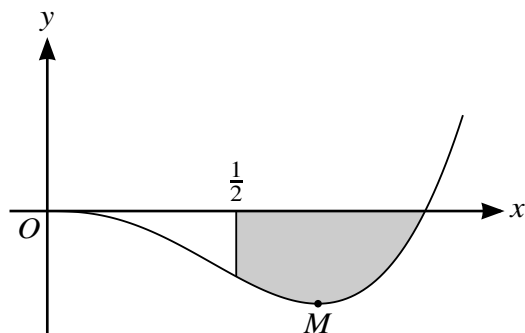
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The diagram shows the curve  $y = x^3 \ln x$ , for  $x > 0$ , and its minimum point  $M$ .

(a) Find the exact coordinates of  $M$ .

[4]

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- 9 The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = e^{3y} \sin^2 2x.$$

It is given that  $y = 0$  when  $x = 0$ .

Solve the differential equation and find the value of  $y$  when  $x = \frac{1}{2}$ . [7]

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A series of 25 horizontal dotted lines for writing.

10 With respect to the origin  $O$ , the points  $A$ ,  $B$ ,  $C$  and  $D$  have position vectors given by

$$\vec{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \vec{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

(a) Find the obtuse angle between the vectors  $\vec{OA}$  and  $\vec{OB}$ . [3]

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The line  $l$  passes through the points  $A$  and  $B$ .

(b) Find a vector equation for the line  $l$ . [2]

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- (c) Find the position vector of the point of intersection of the line  $l$  and the line passing through  $C$  and  $D$ . [4]

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11 Let  $f(x) = \frac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$ .

(a) Express  $f(x)$  in partial fractions. [5]

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(b) Hence show that  $\int_0^2 f(x) dx = \ln 54 - \frac{1}{8}\pi$ . [5]

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**Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Lined area for writing answers, consisting of multiple horizontal dotted lines.

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